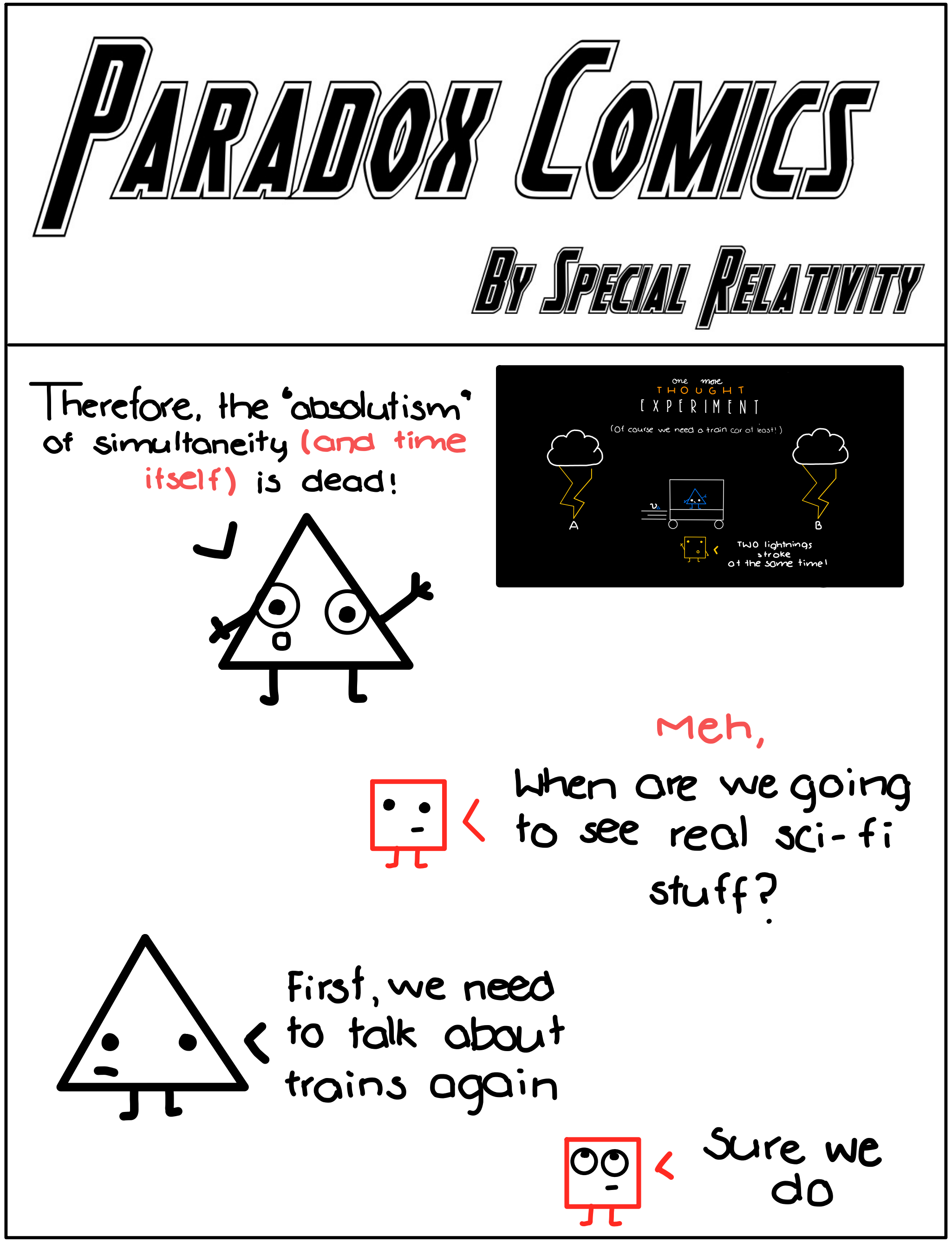
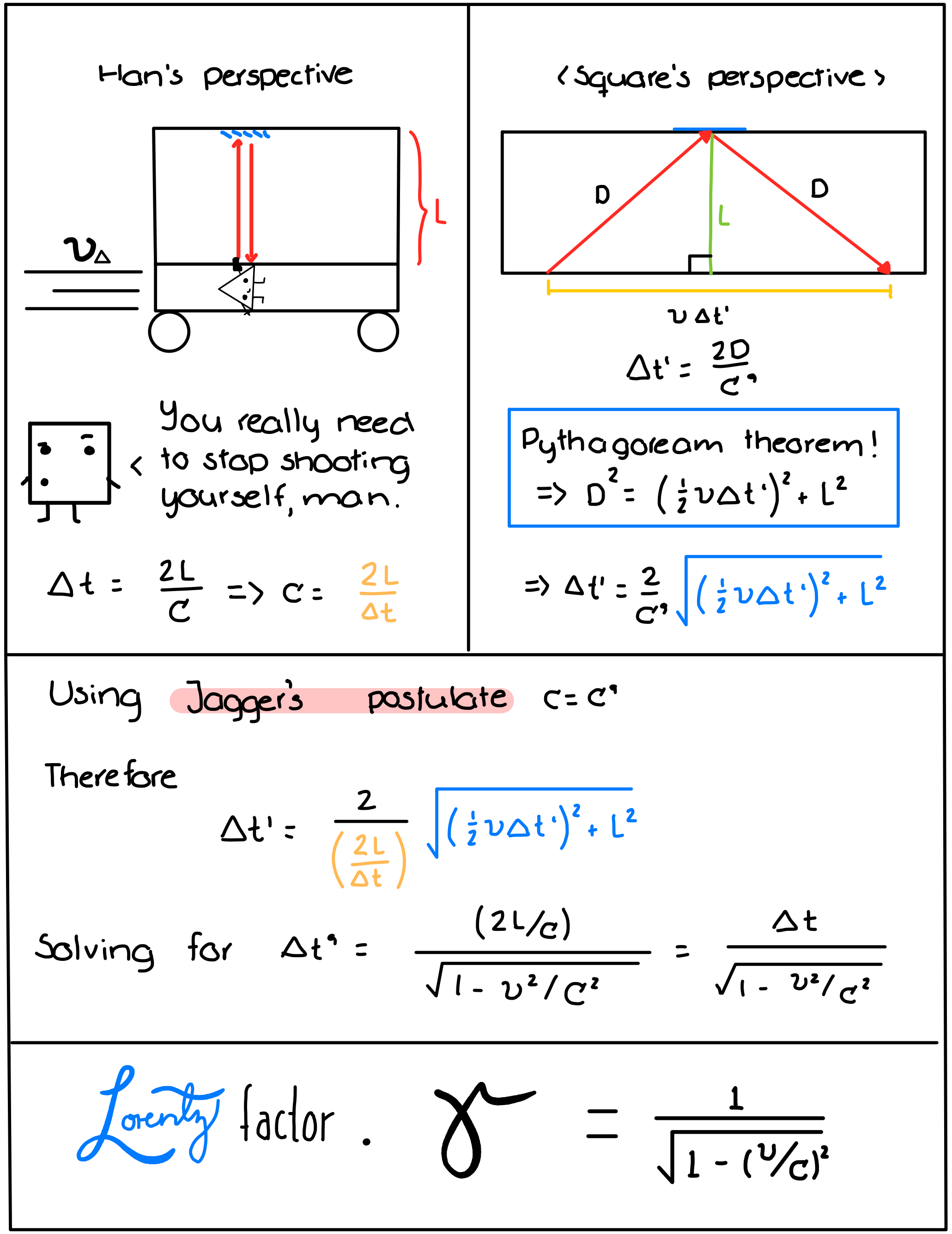
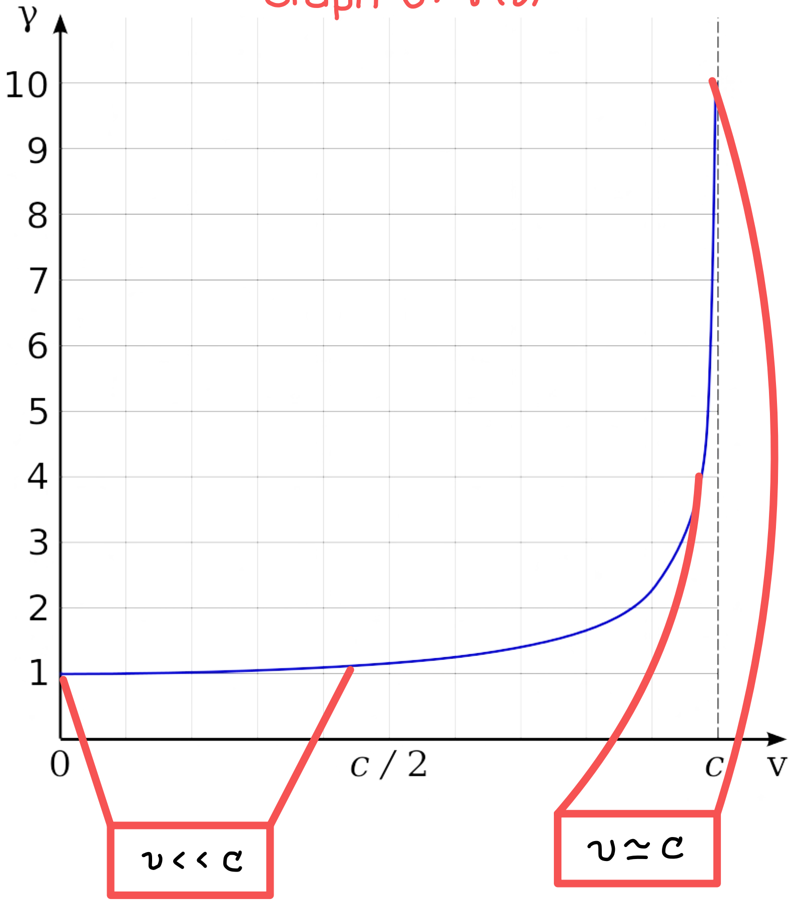

PARADOX COMICS
BY SPECIAL RELATIVITY
Therefore, the 'absolutism' of simultaneity (and time itself) is dead!
one more THOUGHT EXPERIMENT
(Of course we need a train car at least!)
A
B
Two lightnings stroke at the same time!
Meh, when are we going to see real sci-fi stuff?
First, we need to talk about trains again
Sure we do


Han's perspective
$v_\triangle$
L
You really need to stop shooting yourself, man.
$\Delta t = \frac{2L}{c} \Rightarrow c = \frac{2L}{\Delta t}$
‹Square's perspective›
D
D
L
$v\Delta t'$
$\Delta t' = \frac{2D}{c'}$
Pythagorean theorem!
$\Rightarrow D^2 = \left(\frac{1}{2}v\Delta t'\right)^2 + L^2$
$\Rightarrow \Delta t' = \frac{2}{c'}\sqrt{\left(\frac{1}{2}v\Delta t'\right)^2 + L^2}$
Using Jagger's postulate $c = c'$
Therefore
$\Delta t' = \frac{2}{\left(\frac{2L}{\Delta t}\right)}\sqrt{\left(\frac{1}{2}v\Delta t'\right)^2 + L^2}$
Solving for $\Delta t' = \frac{(2L/c)}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}$
Lorentz factor. $\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$

LORENTZ FACTOR $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ $\rightarrow$ $\boxed{\beta = \frac{v}{c}}$

Graph of $\gamma(v)$



$\Rightarrow \beta \simeq 0$ { small number divided by a BIG NUMBER }

Like F1 cars, sonic planes

372.5 km/h   3,500 km/h

$\Rightarrow \beta \simeq 1$ { Similar numbers dividing each other }

Like subatomic particles

$v = 0.99975\,c$

Returning to the previous experiment:

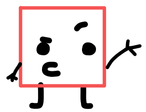
$$(\Delta t)' = \gamma(\Delta t)$$

As $\gamma \geq 1 \Rightarrow (\Delta t)' \geq (\Delta t)$

The moving observer's time lapse $(\Delta t)'$ is longer than the time lapse (Δt) in the 'static' frame of reference.

TIME dilation

The square might claim that the triangle's watch is running too slow.

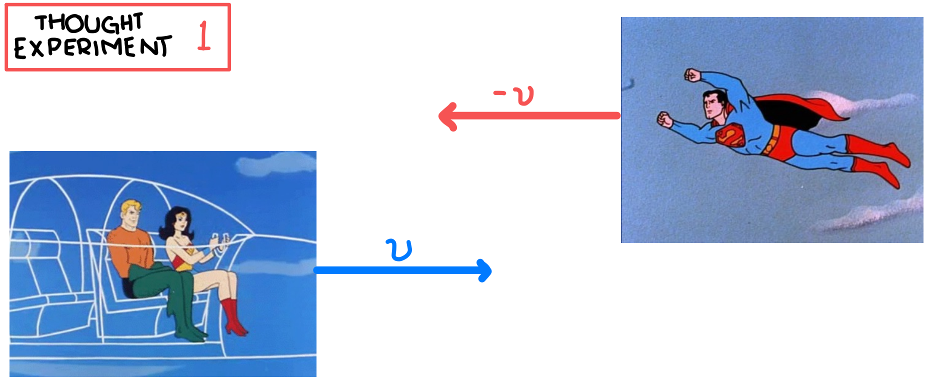


Aha! Then, the time for the square runs faster! So you are violating Dr Dylan's postulate of relativity because physics are different with respect to each observer!

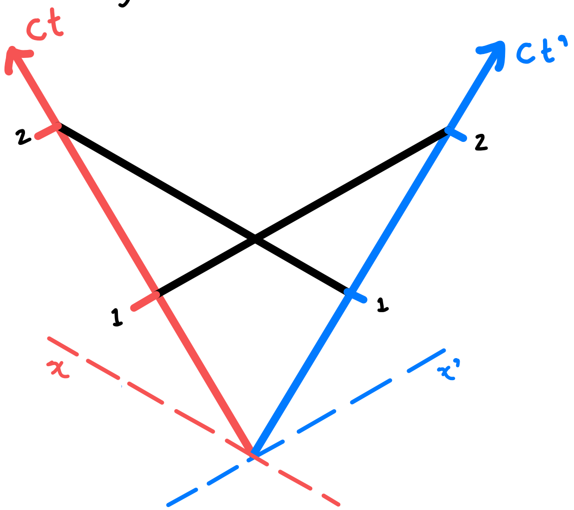
Your time is moving too slow!

Dylan's postulate dictates that there should be some kind of RECIPROCITY between observers because each of them has a VALID INERTIAL FRAME

THOUGHT EXPERIMENT 1



Now, the spacetime diagram would look like this:

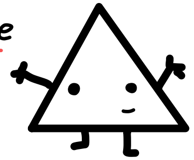


NOTE: both observers perceive that measuring from their frame of reference the other one's time is running slow in the same proportion.

It is not a paradox, it's a matter of perception.

MORAL: Like in the case of SIMULTANIETY the RATE of TIME is dependent of the observer.

We can say that time dilation is in fact a consequence of the failure of simultaneity!

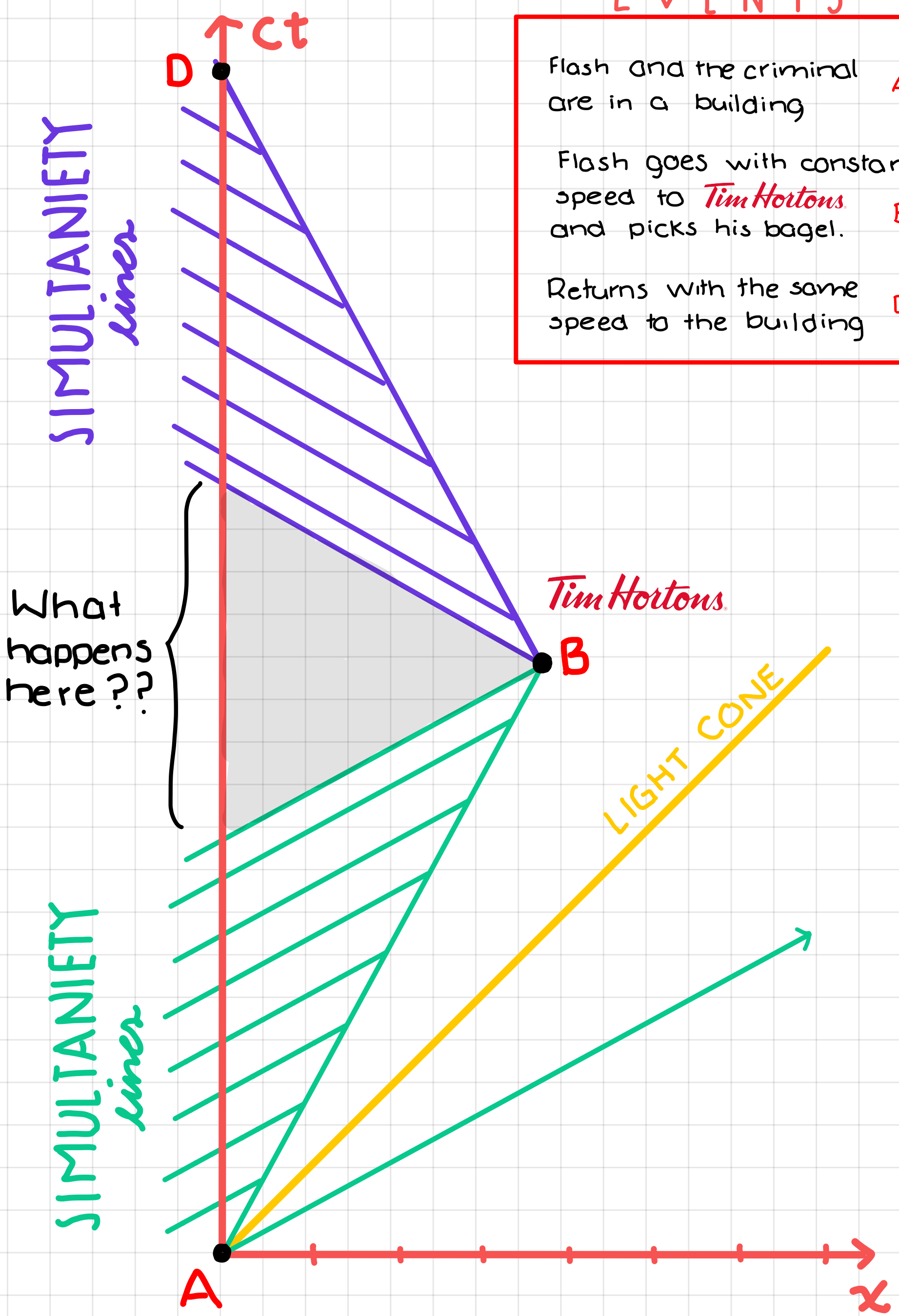


Let's spice things up:

THOUGHT EXPERIMENT 2

< Flash gets a sandwich >

Exercise: Explain with a ST-diagram if this meme makes any sense



EVENTS

Flash and the criminal are in a building	A
Flash goes with constant speed to *Tim Hortons* and picks his bagel.	B
Returns with the same speed to the building	D

Discussion

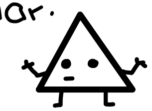
- What are the simultaniety lines telling us ?
- In which ways is this experiment *similar* to the previous one ?
- What can be said about the symmetry of the problem ?
- Do they both (the Flash and the criminal) have a valid reference frame ?
- What happens in the gray area of the ST diagram ?

Let's put numbers to it!

Suppose the Flash runs at $\frac{24}{25}c$ and the Tim Hortons is at $\underbrace{2.2 \times 10^8 s}_{\simeq 7 years}$ (running at that speed).

How older is the criminal when the Flash is back ?

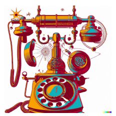
Up until now, we have never made any kind of assertion regarding the speed of light as the "*speed limit of the universe*", or anything similar.



We only have its invariance (*Jagger's postulate*)

Let's suppose that we are able to invent a

‹ TACHYONIC ANTITELEPHONE ›



↓

device that let's us communicate

FTL

"*faster than light*"

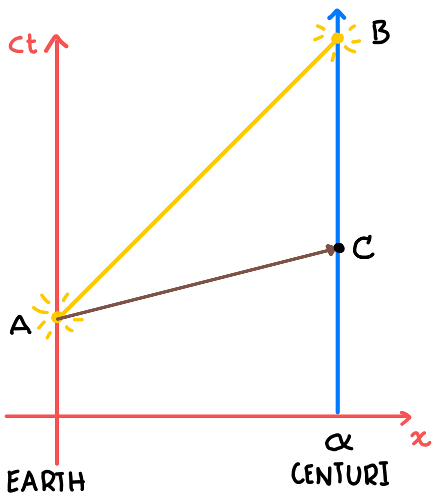
THOUGHT EXPERIMENT:

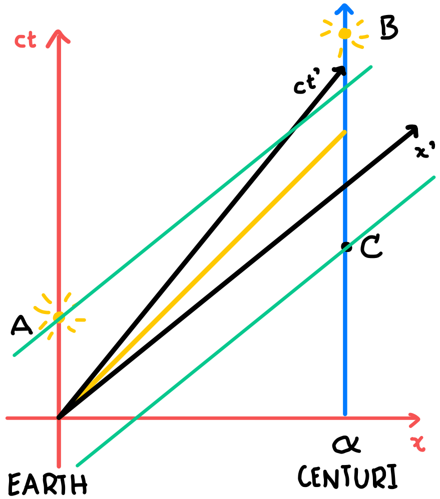
A: Our Sun explotes

B: people on Alpha Centuri see the explotion

→ We send then a FTL message alerting them

C: they receive the message

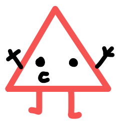




→ Before A, we sent an expedition to Alpha centuri.

= Simultaneity lines from the speceship frame of reference

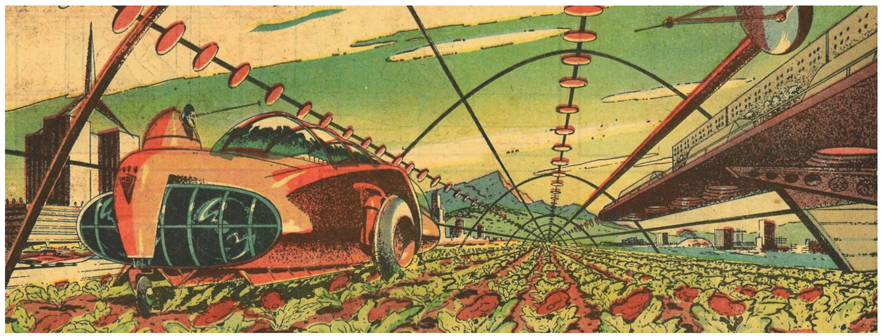
What can we conclude with respect to the simultaneity lines that cross the A & C events?



From the perspective of the people inside the spaceship the message arrive before it is created!

EFFECT precedes CAUSE

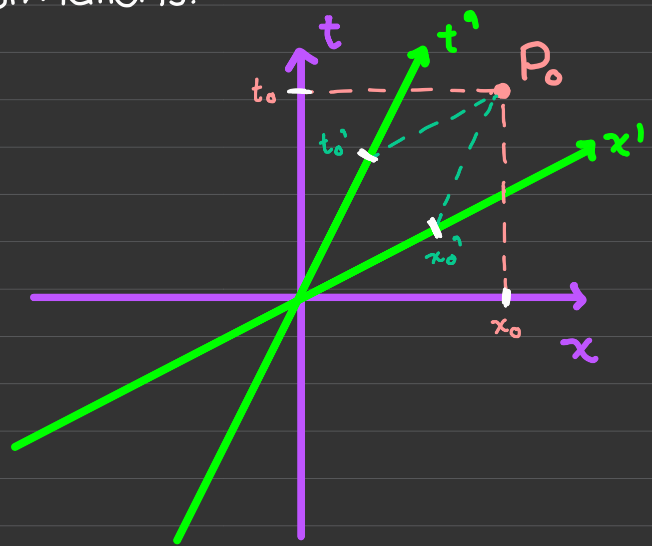
∴ It is not possible to have this type of communication



LORENTZ TRANSFORMATIONS

At this point we have come (geometrically) familiarized with the stretchy & rotational nature of changing one inertial frame of reference to another one.

Now is a good time to establish algebraically a relationship between coordinate systems, in a similar fashion as Galilean transformations.



P_0: (x_0, t_0) $\quad$ (x_0', t_0')

We are able to sense that the proposed transformation must:

- interlace time & space coordinates (with the help of v as currency of exchange
- be able to replicate Galilean transformations when the velocities are small.
- symmetrical with respect to observers.

Prop:

LORENTZ TRANSFORMATIONS

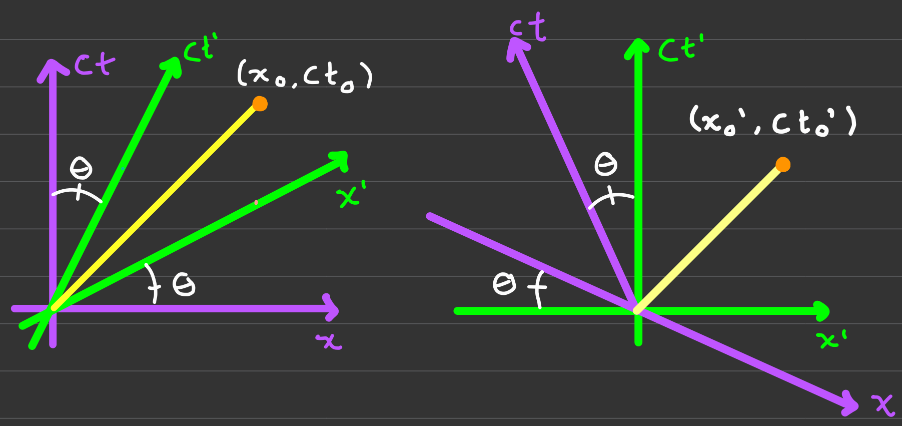
$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

what about y and z? } As the movement is in the x direction, the other spatial coordinates should not be changed

$$\begin{cases} y' = y \\ z' = z \end{cases}$$

Deduction. Let's take an event that is connected to the origin by a light path:



Let's take the galilean transformation and scale it by some unknown factor

$$x' = \alpha(x - vt) \qquad\qquad x = \alpha(x' + vt')$$

same factor!

Now multiplying both of the expressions:

$$x'x = \alpha^2 (x - vt)(x' + vt')$$

$$= \alpha^2 (xx' + xvt' - x'vt - v^2tt')$$

For our event in particular we have that

$$x_0 = ct_0 \qquad \& \qquad x_0' = ct_0'$$

Substituting (and ignoring the nought)

$$x'x = \alpha^2 (xx' + xvt' - x'vt - v^2tt')$$

$$\Rightarrow (ct')(ct) = \alpha^2 (c^2tt' + \cancel{ct'tv} - \cancel{ct'tv} - v^2tt')$$

$$\Rightarrow c^2 \cancel{t't} = \alpha^2 \cancel{t't}(c^2 - v^2)$$

$$\boxed{\therefore \alpha^2 = \frac{c^2}{c^2 - v^2}}$$

$$\Rightarrow \boxed{\alpha^2 = \frac{\cancel{c^2}/\cancel{c^2}}{\frac{c^2 - v^2}{c^2}} = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1}{1 - \beta^2} = \gamma^2}$$

We have found Lorentz factor again!

It is left to the reader to corroborate now the transformation for the time variable.

A "cleaner" version of LT can be stated as:

$$\boxed{\begin{aligned} x' &= \gamma(x - \beta ct) \\ ct' &= \gamma(ct - \beta x) \end{aligned}}$$